Approaching the AWGN Channel Capacity without Active Shaping[1]

Long Duan†, Bixio Rimoldi†, Rüdiger Urbanke‡
ld1@ee.wustl.edu, bixio@ee.wustl.edu, ruediger@research.att.com
†Dept. of Electrical Engineering, Washington University, St. Louis, MO 63130, USA
‡Bell Laboratories, 600 Mountain Ave. Murry Hill, NJ 07974

Abstract — **It is shown that the capacity of the AWGN channel can be approached via a multilevel coding scheme with the output of each encoder mapped into an independent signal constellation. No active shaping is required in this scheme, regardless of the signal-to-noise ratio. Moreover, shifted linear codes can be used.**

I. Introduction

The capacity of the additive white Gaussian noise (AWGN) channel with power constraint P and noise variance N is $C(P,N) \triangleq (1/2)\log(1+P/N)$. To approach this capacity at high signal-to-noise ratios (SNRs), one has to transmit several bits per dimension. A major breakthrough toward this goal was achieved in the early 80s by Ungerböck [1]. Ungerböck showed how to obtain high rate codes by using the *mapping by set partitioning* technique. Since then, this technique has been refined in various ways.

What seems to be common to all current techniques for AWGN channels at high SNRs is the use of a large signal constellation, usually a lattice, and shaping. The purpose of this paper is to show that neither one is required to approach capacity. The idea is to do multilevel coding but map the output of each (possibly binary) encoder into a signal constellation (possibly antipodal) which is independent from the signal constellations used by other encoders. In this way we completely decouple each level and we let the central limit theorem and the channel noise ensure that the marginal distributions (at the receiver) be approximately Gaussian, as required to approach the capacity.

II. A Multilevel Coding Scheme

Consider putting a mapping $f : \mathcal{X}_1 \times \mathcal{X}_2 \times \cdots \times \mathcal{X}_M \to \mathbf{R}$ in front of the AWGN channel with power constraint P and noise variance N as shown in Fig. 1. We pick the mapping in such a way that

$$C(P,N) - \max_{P_X : E\{X^2\} \le P} I(X_1, \cdots, X_M; Y) \le \epsilon \qquad (1)$$

where ϵ is some small positive number. Notice that in order to satisfy (1), one only has to make the output distribution Y sufficiently close to that of a Gaussian random variable with variance $P+N$. By the central limit theorem, this can be achieved via independent inputs and $f(X_1, X_2, \cdots, X_M) = \sum_{i=1}^{M} X_i$.

Hence assume that the inputs $X_1, X_2, \cdots, X_M$ are mutually independent and that X_i takes values in the finite set $\mathcal{X}_i$. By the chain rule of mutual information, we have $I(X_1, \cdots, X_M; Y) = \sum_{i=1}^{M} I(X_i; Y | X_1, \cdots, X_{i-1}) = \sum_{i=1}^{M} I(X_i; Y, X_1, \cdots, X_{i-1})$, where the last equality follows from the independence of $X_1, X_2, \cdots, X_M$. This means that we can approach the mutual information $I(X_1, \cdots, X_M; Y)$ via M independents codes and doing stripping at the decoder. More explicitly, code i is designed to achieve $I(X_i; Y, X_1, \cdots, X_{i-1})$ on the channel with input $\mathcal{X}_i$ and output $\mathcal{Y} \times \mathcal{X}_1 \times \cdots \times \mathcal{X}_{i-1}$.

If each alphabet $\mathcal{X}_i$ is a Galois field, then Gabidulin's theorem [2] tells us that the rate $I(X_i; Y, X_1, \cdots, X_{i-1})$ associated with level i can be approached via a shifted linear code.

In conclusion, one may approach the capacity of an AWGN channel via some number M of shifted linear codes, an independent signal constellation for each encoder, and without use of shaping.

Fig. 2 shows simulation results using rate-1/2 turbo encoder at each input level. The output of each encoder is fed into an antipodal modulator.

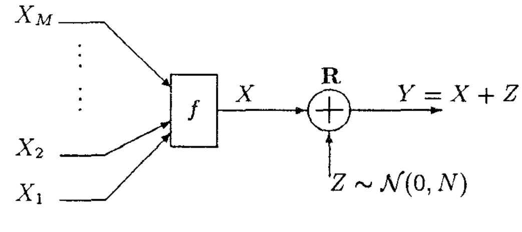

Figure 1: The multiple-input channel obtained by cascading the function f and the AWGN channel.

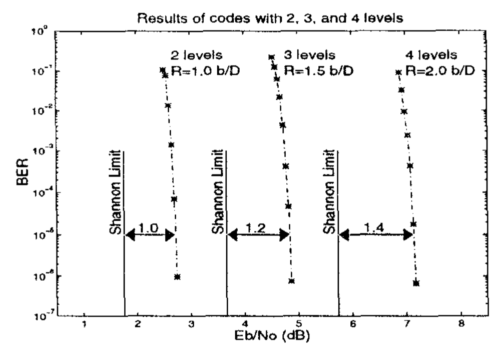

Figure 2: The performances of the multilevel codes with rate-1/2 turbo codes at each input level.

References

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[2] E. M. Gabidulin, "Bounds for the probability of decoding error when using linear codes over memoryless channels," *Problemy Peredači Informacii (Problem of Information Transmission)*, vol.3, no.2, pp. 55-62, 1967.

[1] Supported by National Science Foundation grants NCR-9357689 and NCR-9304763.

